Exam Review: Ch 4-5 MDM 4U0
Answer Section

MULTIPLE CHOICE

1. ANS: C REF: Knowledge & Understanding OBJ: Section 4.1
   LOC: CP1.02 TOP: Fundamental counting principle
2. ANS: B REF: Knowledge & Understanding OBJ: Section 4.1
   LOC: CP1.02 TOP: Fundamental counting principle
3. ANS: D REF: Knowledge & Understanding OBJ: Section 4.3
   LOC: CP1.05 TOP: Permutations with some identical items
4. ANS: C REF: Knowledge & Understanding OBJ: Section 4.3
   LOC: CP1.05 TOP: Permutations with some identical items
5. ANS: C REF: Knowledge & Understanding OBJ: Section 4.4
   LOC: CP1.07 TOP: Pascal's triangle
6. ANS: C REF: Knowledge & Understanding OBJ: Section 4.5
   LOC: CP1.07 TOP: Applying Pascal's method

SHORT ANSWER

7. ANS:
   Enzo has 144 ways to choose his courses since he can choose his major courses in $4 \times 3 \times 2$ ways and his options in 3×2 ways. By the multiplicative rule, he has $24 \times 6 = 144$ ways in which he can rank the courses.

8. ANS:
   You can roll a sum of 6 or a sum of 10 in 8 ways.

9. ANS:
   There are 12 ways to choose a 4, a 9, or a jack.

10. ANS:
    For each question there are two choices. Applying the multiplicative (fundamental) counting principle, there are $2^8 = 256$ ways a student could answer the test.

11. ANS:
    There are 6 760 000 postal codes possible.

12. ANS:
    There are 4 100 625 postal codes possible.

13. ANS:
    There are 3 175 524 postal codes possible.

14. ANS:
    There are 2520 such arrangements.

15. ANS:

a) 604 800
b) $7.602\ 692\ 257 \times 10^{14}$
c) $1.157\ 655\ 162 \times 10^{16}$

16. ANS:
a) 8!
b) 11!
c) 7!

17. ANS:
Linda can arrange the books in 5! = 120 ways.

18. ANS:
There are six students making presentations between John's and Linda's. These six students can appear in ${}_6P_6 = 720$ different orders.

19. 24 20. 24 21. 3360 22. 20 160 23. $t_{7,3}$ 24. row 5

PROBLEM

25. ANS:
Direct method
The letter *t* can be any of the five letters in the arrangement. If *t* is the first letter, there are six choices left for the second letter, five choices for the third letter, four for the fourth, and three for the fifth. Using the multiplicative counting principle, there are $6 \times 5 \times 4 \times 3 = 360$ arrangements with *t* as the first letter. There are the same number of arrangements with the letter *t* in each of the other four positions. Thus, the total number of five-letter arrangements that include the letter *t* is $5 \times 360 = 1800$.

Indirect method
Find the total number of five-letter arrangements and subtract those that do not contain the letter *t*. The total number of five-letter arrangements is $7 \times 6 \times 5 \times 4 \times 3 = 2520$. In arrangements without the letter *t,* there are six choices for the first letter. For the second letter, there are five choices left; for the third, four; for the fourth, three; and for the fifth, two. Applying the multiplicative counting principle, the number of arrangements without the letter *t* is $6 \times 5 \times 4 \times 3 \times 2 = 720$. Therefore, the number of arrangements with the letter *t* is $2520 - 720 = 1800$.

26. ANS:
a) There are 10 choices for each of the two numbers and 26 for each of the letters. The total number of licences possible using this system is $10 \times 10 \times 26 \times 26 \times 26 = 1\ 757\ 600$.
b) If *1*s, *0*s, *I*s, and *O*s are not used, the number of possible licences is $8 \times 8 \times 24 \times 24 \times 24 = 884\ 736$. The number of licences you are likely to need is $130\ 000 \times 110\% = 143\ 000$, so you will not have to change the system this year.

27. ANS:
a) ${}_7P_7 = 5040$
b) ${}_6P_6 = 720$
c) Treat *r* and *s* as a unit. This pair can be arranged in ${}_6P_6$ ways with the remaining letters. The pair itself can be arranged as *rs* or *sr*, so there is a total of ${}_6P_6 \times 2 = 1440$ arrangements with *r* and *s* adjacent.

d) There are ten ways in which a group of three letters can be formed with the letters *r* and *s* separated by one of the other five letters. Consider this group as a unit. It can be arranged with the remaining four letters in ${}_5P_5$ ways. Thus, there are ${}_5P_5 \times 10 = 1200$ arrangements in which the letters *r* and *s* are separated by only one letter.

e) First, find the number of arrangements in *r* and *s* are adjacent *and a* and *n* are also adjacent. As in part c), consider each pair of letters as a unit. The pairs can be arranged with the remaining three letters in ${}_5P_5$ ways. Each pair can itself be arranged in two ways, so the number of permutations with both *r* and *s* adjacent and *a* and *n* adjacent is $5! \times 2 \times 2 = 480$. From part c), there are a total of 1440 permutations with *r* and *s* adjacent, so the number of these permutations that have *a* and *n* separate is 1440 – 480 = 960.

28. ANS:

a) Since the three numbers are different, the number of lock combinations is ${}_{30}P_3 = 24\,360$.

b) On average, you would try half the combinations before finding the right one. At half a minute each, the average time required would be $\frac{24\,360}{2 \times 0.5 \times 60} = 406 \text{ h}$.

29. ANS:

Consider the chair and vice-chair as a unit. This pair can be arranged with the remaining six members in ${}_7P_7 = 5040$ ways. For each of these ways, the chair could be either on the left or the right of the vice-chair. Therefore, there is a total of $2 \times 5040 = 10\,080$ ways in which the students' council could pose for the photograph.

30. ANS:

There can be no more than a total of four white tiles and gold tiles. Since there must be at least one white tile and fewer white tiles than gold tiles, the only possible choices are one white tile with either two gold tiles plus nine blue tiles or three gold tiles plus eight blue tiles. The total number of border arrangements for these two cases is $\frac{12!}{2!9!} + \frac{12!}{3!8!} = 2640$.

31. ANS:

The number of direct routes Gisela can follow is the number of different orders in which she can choose to travel the eight blocks south and the six blocks east. Therefore the formula for permutations with some identical items can be applied. The number of routes is $\frac{14!}{8!6!} = 3003$.

32. ANS:

a) The maximum number of intersections occurs if each straw overlaps all of the other straws in the pile. Start with a single straw. When a second straw is added it can overlap the first at a maximum of one point. A third straw can overlap the first two at a maximum of one point each, making a total of 1 + 2 = 3 intersections. Similarly, a fourth straw can add a maximum of three intersections for a total 1 + 2 + 3 = 6, and the fifth straw adds a maximum of four more intersections for a total of 1 + 2 + 3 + 4 = 10.

b) These totals correspond to the entries on the third diagonal of Pascal's triangle. The total for five straws is the value of $t_{5,3}$.