Unit 1: Counting & Probability

In this unit, you will learn about probability and how it is used to predict an outcome of an experiment or the likelihood of events occurring.

1.1 The Language of Probability

Before we can start calculating any sort of complicated probabilities, we need to all be on the same page with the language we are using…

A. What is probability?

Probability is the branch of mathematics that attempts to predict the likelihood of events occurring.

For instance,

- How likely is it that it will snow tomorrow?
- What are the chances that you will get a mark of 80% in this course?
- What are the odds that the Toronto Maple Leafs will win the cup?

B. The 3 types of probability

1. Empirical Probability (a.k.a. experimental probability)

- Based on direct observation or experiment

Example:

2. Theoretical Probability (aka classical or a priori)

- Based on mathematical analysis

Example:

3. Subjective Probability

- Based on informed guesswork

Example:

C. Probability Definitions:

- **Sample Space** – the set of all possible outcomes in a probability experiment. Sample spaces can be **discrete** or **continuous.**
  - **Discrete Sample Space:** outcomes can be counted

    Example:

  - **Continuous Sample Space:** outcomes that can be measured

    Example:

  - This unit deals mostly with discrete sample spaces

- **Trial** – a step in a probability experiment in which an outcome is produced and the results tallied
- **Outcome** – a possible result, a component of an experiment

The probability of an event A is given by: $P(A) = \frac{n(A)}{n(S)}$, where

P(A) = Probability of event A
n(A) = # outcomes of A
n(S) = # of outcomes in sample space

Note: If P(A) = 0, event A is **impossible**, if P(A) = 1, event A is **certain**

Example 1: List what is in the sample space of rolling a standard dice:

Example 2: Use the table to list the sample space of rolling 2 standard dice

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Example 3: What is the probability of rolling a sum of "7" with two standard dice?

Example 4: What is the probability of drawing a King **or** a Black Jack from a standard deck of cards?

Example 5: A coin is tossed 3 times. What is the probability of getting at least 2 heads? (Use a tree diagram).

Example 6: What is the probability of rolling a number less than five using a die?

Example 7: What is the probability of rolling an even number using a die?

Example 8: Find the probability of rolling an even prime number using a die.

Example 9: Find the probability of rolling either an even prime or an odd prime.

D. The Complement Property:

The sum of the probabilities of all possible events is always equal to 1. This means that $P(A) + P(A') = 1$. $P(A')$ is the probability of an event **not** occurring (read A prime). A' is the complement of A.

Rearranged, $P(A') = 1 - P(A)$ or
$P(A) = 1 - P(A')$

Example 6: What is the probability that a randomly drawn integer between 1 and 40 is **not** a perfect square?

Example 7: Find the probability of NOT drawing a face card (*K*, *Q*, *J*) from a well-shuffled deck.

Example 8: Experimental probability
A single die is rolled 60 times in a row and the number appearing on the up face is observed.
The results are recorded below.

Number on the die	1	2	3	4	5	6
Occurrences	7	12	11	10	8	12

a) Find the experimental probability of rolling a 5 using the die

b) Find the experimental probability of rolling a 4 using the die

c) Find the experimental probability of rolling a 7 using the die

d) Find the experimental probability of rolling a number less than 7 using the die

e) Find the theoretical probability of parts a-d above and compare them with the experimental probabilities

f) How do you think the experimental probability would change if the number of trials were 600 instead of 60?