Name: _____ MDM 4U1 Date: _____

1.5 Learning to Count

Counting is one of the first mathematical things you learn as a child. When we're dealing with probability it can get pretty complicated though. Here you'll lean some strategies to calculate the number of outcomes in large sets.

How many ways can you rearrange the letters of the words below?

a) CAT

b) SONG

c) JOKER

d) HANGAR

e) BOSCO

A. Factorial

Factorial notation is a way of expressing the number of ways you can arrange *n* objects…

$$n! = n\times(n-1)\times(n-2)\times \ldots\ 3\times2\times1$$

Most scientific calculators have a factorial button so you don't have to punch in all the numbers!

$5! =$	$2! =$
$4! =$	$1! =$
$3! =$	$0! =$

Example 1: Simplify the following

a) $\frac{19!}{17!}$ b) $\frac{12!}{7!5!}$ c) $\frac{n!}{(n-2)!}$

Example 2: There are 10 students in the cafeteria. How many different ways can they arrange themselves in a line?

Example 3: How many ways are there to draw 2 cards out a 52 card deck

a) with replacement

b) without replacement

Example 4: In how many ways can the seven members of the math club sit

a) In a row

b) In a row with the president in the middle

c) In a row with Amy and Bill side by side

d) In a row with Amy and Bill NOT side by side

Example 5: In how many ways can the word LAUGHTER be arranged?

Example 6: In how many ways can the word TORONTO be arranged?

Example 7: Mr. Notten has 10 different pictures of cool space things that he wants to hang on his wall. In how many ways can he hang **four** of them in a row?

B. Permutations

An ordered arrangement of objects selected from a set. Choose *r* objects of a set of *n* total objects

$$P(n,r) = \frac{n!}{(n-r)!}$$

Note: sometimes $P(n,r)$ is written as ${}_nP_r$
This is also a button on most scientific calculators!

Example 8: Solve this question again but with the permutation formula!
Mr. Notten has 10 different pictures of cool space things that he wants to hang on his wall. In how many ways can he hang **four** of them in a row?

Example 9: There are 15 players on the school baseball team. How many ways can the coach complete the nine-person batting order?

Example 10: In how many ways can a class of 23 students select a president, vice president and secretary if… a)There are no restrictions? b) Michael does not wish to be selected?

c) What is the **probability** Abe, Bob and Carol are selected if all 23 students are running?

Example 11: The letters B-O-S-C-O are placed in a hat. What is the probability that when you draw them they will spell BOSCO?

Example 12: A 12-volume encyclopedia is to be placed on a shelf.

a) How many incorrect arrangements are there?

b) If they are placed randomly, what is the probability they are placed correctly?

Example 13: 5 players are playing a card game with a standard deck of cards. Each player is dealt a card. How many ways can the cards be dealt?

Example 14: A composer named Schoenberg pioneered a technique for composing music where each of the 12 tones in an octave is used only once. Each sequence is called a tone row. How many tone rows are possible?

Challenge: Solve for n

a) $\frac{(n+3)!}{(n+2)!} = 10$

b) ${}_nP_5 = 13\,{}_nP_4$

Homework: Textbook pg 255# 1-8, 11, Challenge: 14-15

1.6 More Counting: Does Order Matter?

Sometimes when your picking from a group, the order in which you pick doesn't matter! We can't really use permutations for this. Check out the example below:

In a card game, you are dealt a hand of 5 cards. How many possible hands can you be dealt in a 52 card deck?

Does order matter??

A. Combinations

A collection of chosen objects for which order doesn't matter. Choose *r* objects from a set of *n* total objects

$$C(n,r) = \frac{n!}{(n-r)!\,r!}$$

Note: sometimes $C(n,r)$ is written as ${}_nC_r$ OR even better: $\binom{n}{r}$

This is also a button on most scientific calculators!

Combinations are closely related to permutations: $C(n,r) = \frac{P(n,r)}{r!}$

Example 1: There are 12 players on the school basketball team. Only 5 players can start the game.

a) How many different groups of starting players can the coach make? *Does order matter?*

b) How many different lineups can the coach make (positions matter)! *Does order matter?*

Example 2: In how many ways can 6 people be selected from a group that consists of 4 adults and 8 children. The group selected must contain **at least** 2 adults.

Example 3: In a game where you are dealt 5 cards from a deck of 52,
a) How many ways can you be dealt a royal flush (10-J-Q-K-A of the same suit)?

b) How many ways can you be dealt a straight (5 cards in order)?

b) How many ways can you be dealt four of a kind?

c) What is the probability of getting each of these?

Unit Task

Before we can start calculating any sort of complicated probabilities, we need to all be on the same page with the language we are using…

Unit assignment – make a game that LOOKS like it's fair but isn't.
Show the probabilities
Work in partners
Try it out! Record who wins.